ANALYSIS OF THE "TAILORED" CONTACT SURFACE REGION IN A SHOCK TUBE

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A simple method is presented for the computation of the initial parameters of the gas, the working time, and the optimal relationship between the lengths of the high-pressure chamber and the low-pressure channel of a shock tube in the "tailored" contact surface region which permits a substantial increase in the working time.

The "tailored" contact surface region proposed for shock tubes in [1] permits an increase in the working time, an improvement in the homogeneity of the working gas parameters, and turns out to be useful in many cases, for instance, in aerodynamics and aerophysics investigations in hypersonic shock tubes equipped with nozzles of large linear dimensions, in investigations on chemical kinetics, and in the operation of quantum and MHD generators [2, 3].

The x vs t flow diagram in a shock tube of optimal length in the "tailored" interface region is presented in Fig. 1 in the version when the reflected shock wave, the rarefaction wave tail, and the bow reflected rarefaction wave meet at a single point (B).

Distinctive methods for computing the "tailored" interface region, which require successive determination of the gas parameters in different flow domains (Fig. 1), are presented in [4, 5].

In addition to the formulas contained in [4, 5], let us present a simple expression which will permit computation of the ratio T_4/T_1 between the initial temperatures of the working and driving gases with the ratios γ_1, γ_4 between the specific heats and the known molecular weights μ_1, μ_4, by using just the given incident shock-wave intensity p_2/p_1.

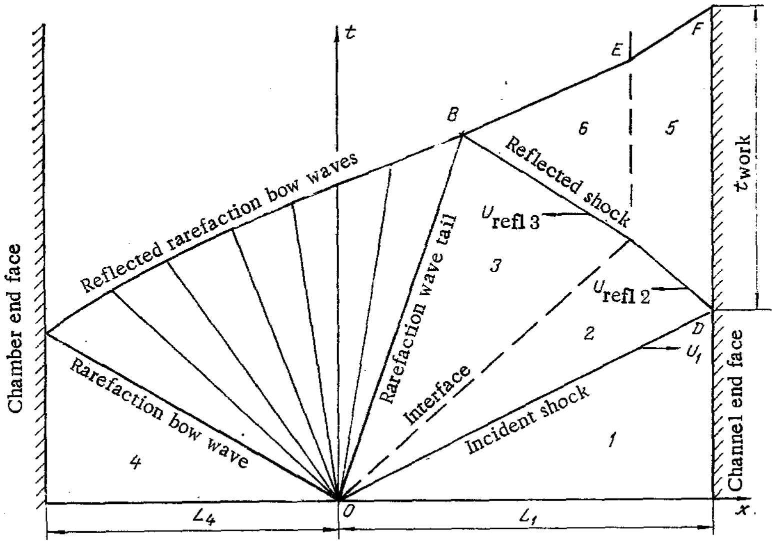

Fig. 1. x vs t diagram of perturbation propagation in a shock tube of optimal length in the "tailored" interface mode.

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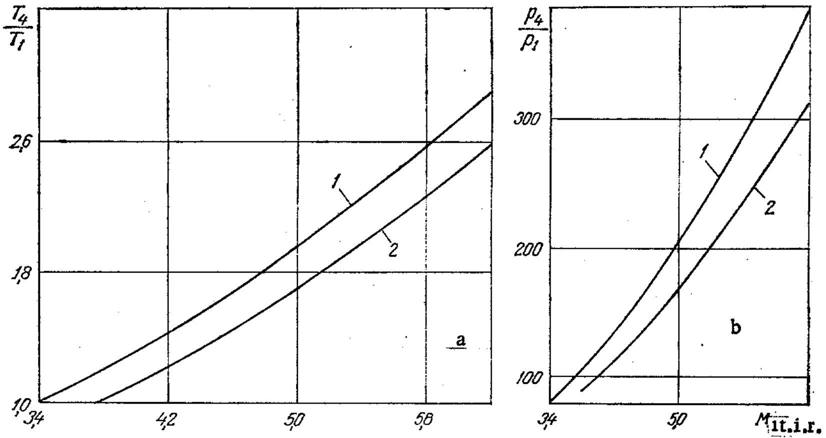

Fig. 2. The ratios T_{41} between the initial temperatures of the driving and working gases (a) and p_{41} between the initial pressures of the working and driving gases (b) as a function of the number $M_{1t.i.r.}$ of the shock in the "tailored" interface region: 1) helium, $\gamma_4 = 5/3$; air, $\gamma_1 = 7/5$; 2) helium, $\gamma_4 = 5/3$; argon $\gamma_1 = 5/3$.

Taking account of the condition that the pressures are equal in regions 5 and 6 (Fig. 1), the condition that changes in the stream velocity on the reflected shock are equal in the working and driving gases, and using the known relationships of elementary shock-tube theory [6], we obtain

$$T_{41} = \frac{\mu_{41}(\gamma_4 - 1)}{(\gamma_1 - 1)(\alpha_1 p_{21} + 1)} \left\{ \sqrt{\frac{[p_{21}(\alpha_1\alpha_4 + 2\alpha_4 + 1) + \alpha_1 - \alpha_4](p_{21} + \alpha_1)}{\alpha_1(\alpha_1 + 2) + 1}} + \sqrt{\beta_4}(p_{21} - 1) \right\}^2, \tag{1}$$

where

$$\alpha = \frac{\gamma + 1}{\gamma - 1}; \quad \beta = \frac{\gamma - 1}{2\gamma}.$$

Here and henceforth, the number subscripts will denote the domains with homogeneous gas parameters (Fig. 1) in conformity with elementary shock-tube theory. The double subscript denotes the ratio of the gas parameters in definite flow domains, for instance, $p_{21} = p_2/p_1$.

The initial ratio p_{41} between the pressures on the diaphragm for a given ratio p_{21} and the ratio T_{41} found from (1) can be calculated by means of a known formula written in the form

$$p_{41} = p_{21} \left[1 - (\gamma_4 - 1) \sqrt{\frac{\mu_{41}}{2\gamma_4(\gamma_1 - 1)}} \cdot \frac{p_{21} - 1}{\sqrt{T_{41}(\alpha_1 p_{21} + 1)}} \right]^{-\frac{1}{\beta_4}}. \tag{2}$$

Let us note that (1) and (2) determine the initial gas parameters in a constant section shock tube with a blind end face.

Graphs representing the dependence of the relative initial gas parameters T_{41}, p_{41} on the incident shock Mach number M_1 in the "tailored" interface region (t. i. r.) are presented in Fig. 2 for combinations of the gases helium—air and helium—argon.

A computation of the gas parameters behind a reflected shock by means of the relationships of elementary shock-tube theory for the initial working and driving gas parameters found by means of (1) and (2) shows that a high value of the pressure "recovery factor" p_{54}, close to 1 and varying slightly with the increase in the shock number M_1, is characteristic for the "tailored" interface region.

In addition to the formulas derived for strong shocks [4], let us present an expression for the ratio between the gas parameters on the interface behind the incident shock in the "tailored" interface region, which is valid for arbitrary intensity shocks.

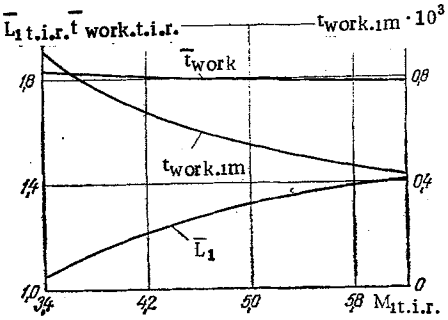

Fig. 3. The parameters $\bar{L}_{1t.i.r.}$, $\bar{t}_{work.\ t.i.r.}$ $t_{work.\ im}$ (sec) characterizing the linear dimensions and working time of a shock tube of optimal length in the "tailored" interface region, helium—air case $\gamma_4 = 5/3$, $\gamma_1 = 7/5$.

The ratio a_{32} between the speeds of sound can be represented as

$$a_{32} = M_{23} = \sqrt{\frac{\gamma_4(\gamma_4+1)}{\gamma_1(\gamma_1+1)}}\sqrt{\frac{1+1/\alpha_4 p_{52}}{1+1/\alpha_1 p_{52}}}. \tag{3}$$

Taking account of the smallness of and the small difference between the quantities $1/\alpha_4 p_{52}$ and $1/\alpha_1 p_{52}$, let us write an approximate expression for the ratio a_{32} in the form

$$a_{32} \simeq \sqrt{\frac{\gamma_4(\gamma_4+1)}{\gamma_1(\gamma_1+1)}}\left(1+\frac{\alpha_1-\alpha_4}{2\alpha_1\alpha_4 p_{52}}\right). \tag{4}$$

Manipulations analogous to those performed for the ratio a_{32} result in the following:

$$\frac{M_{refl_3}}{M_{refl_2}} = \frac{|U_{refl\,3}|+u_2}{|U_{refl\,2}|+u_2}a_{23} = \sqrt{\frac{\alpha_4\beta_4 p_{52}+\beta_4}{\alpha_1\beta_1 p_{52}+\beta_1}} \simeq \sqrt{\frac{\alpha_4\beta_4}{\alpha_1\beta_1}}\left(1+\frac{\alpha_1-\alpha_4}{2\alpha_1\alpha_4 p_{52}}\right), \tag{5}$$

$$T_{32} = a_{32}^2\mu_{41}\gamma_{14} \simeq \mu_{41}\frac{\gamma_4+1}{\gamma_1+1}\left(1+\frac{\alpha_1-\alpha_4}{\alpha_1\alpha_4 p_{52}}\right), \tag{6}$$

$$\rho_{32} = T_{23}\mu_{41} \simeq \frac{\gamma_1+1}{\gamma_4+1}\left(1-\frac{\alpha_1-\alpha_4}{\alpha_1\alpha_4 p_{52}}\right), \tag{7}$$

$$(\rho a)_{32} = \rho_{32}a_{32} \simeq \sqrt{\frac{\alpha_1\beta_1}{\alpha_4\beta_4}}\left(1-\frac{\alpha_1-\alpha_4}{2\alpha_1\alpha_4 p_{52}}\right), \tag{8}$$

$$\frac{U_{refl\,3}}{U_{refl\,2}} = 1+\left(\frac{|U_{refl\,3}|+u_2}{|U_{refl\,2}|+u_2}-1\right)\left(1+\frac{u_2}{|U_{refl\,2}|}\right) \simeq 1+\frac{\gamma_1(\gamma_4-\gamma_1)}{\gamma_1^2-1}\left[1+\frac{2}{(\gamma_1+1)p_{52}}-\frac{3\gamma_1-1}{2\gamma_1(\gamma_1-1)M_1^2}\right], \tag{9}$$

where ρ is the density, u is the coflow velocity, U_{refl} is the velocity of the reflected shock.

Formulas (4)-(9) show that the ratios considered (a_{32} etc.) in the "tailored" interface region are independent in the case $\gamma_1 = \gamma_4$, while they are slightly dependent in the case $\gamma_1 \neq \gamma_4$, on the incident shock number M_1. Thus, for the helium—air combination ($\gamma_4 = 5/3$, $\alpha_4 = 4$, $\beta_4 = 1/5$, $\gamma_1 = 7/5$, $\alpha_1 = 6$, $\beta_1 = 1/7$) under the condition $T_{41} \geq 1$, we have $M_1 \geq 3.4$, $p_{52} \geq 5.46$, $(\alpha_1-\alpha_4)/\alpha_1\alpha_4 p_{52} \leq 0.015$ so that we can consider the ratio between the gas parameters on the interface to be independent of the shock and we can set

$$a_{32} = M_{23} = 1.15;\ \frac{M_{refl_3}}{M_{refl_2}} = 0.97;\ T_{32} = 0.153;\ \rho_{32} = 0.91;\ \frac{U_{refl_3}}{U_{refl_2}} = 1.39;$$
$$(\rho a)_{32} = 1.034.$$

The formula

$$a_{65} = \sqrt{\frac{(\gamma_4-1)\gamma_4}{(\gamma_1-1)\gamma_1}}\cdot\sqrt{\frac{p_{52}+\alpha_4}{p_{52}+\alpha_1}} \tag{10}$$

is valid for the ratio a_{65} between the speeds of sound on the interface behind the reflected shock in the "tailored" interface region, and it does not permit the quantity a_{65} to be considered independent of the intensity p_{52} in the case $\gamma_1 \neq \gamma_4$.

Taking account of the influence of the number M_1 on the ratio a_{65} can be done for a selected range of M_1 numbers by introducing the mean ratio $p_{52\,m}$, since even for moderate numbers M_1 the intensity p_{52} grows slightly with the increase in the number M_1.

Thus, taking the ratio $p_{52\,m} = 6.25$ in the helium—air case with $T_{41} = 1$-3, $M_1 = 3.4$-6.27, $p_{52} = 5.46$-7.05, we will have $a_{65} = 1.3$; $T_{65} = 0.196$; $\rho_{65} = 0.705$; $(\rho a)_{65} = 0.916$.

The relationships (1), (4)-(10) permit approximate expressions to be obtained for the computation of the optimal lengths L_1, L_4 and working times t_{work} in the "tailored" interface region by means of a given shock number M_1.

Introducing the relative length $\bar{L}_1 = L_1/L_4$ of the low-pressure channel and the dimensionless time $\bar{t}_{\text{work}} = (a_4/L_4)\, t_{\text{work}}$, let us write

$$\bar{L}_{1\,\text{t.i.r.}} \simeq \bar{x}_{\text{B}} \frac{(\gamma_1+1)^2}{(\gamma_4+1)(3\gamma_1-1)}\left(1-\frac{3-\gamma_1}{(3\gamma_1-1)M_1^2}\right) + \bar{t}_{\text{B}} M_1 a_{14} \frac{2(\gamma_1\gamma_4-1)}{(\gamma_4+1)(3\gamma_1-1)}\left[1+\frac{\alpha_1(3-\gamma_1)}{2(3\gamma_1-1)M_1^2}\right], \tag{11}$$

$$\bar{t}_{\text{work.t.i.r.}} \simeq \bar{t}_{\text{B}} + \frac{\bar{L}_{1\,\text{t.i.r.}}\, a_{41}}{\sqrt{\frac{2(\alpha_1+2)}{\alpha_1(\alpha_1+1)}p_{21}}}\left[1-\frac{\alpha_1^3+\alpha_1^2-6\alpha_1-4}{4\alpha_1(\alpha_1+2)p_{21}}\right] \times\left\{1-\frac{1}{M_1}\sqrt{\frac{2(\alpha_1+2)p_{21}}{\alpha_1(\alpha_1+1)}}\left[1+\frac{\alpha_1^3+\alpha_1^2-6\alpha_1-4}{4\alpha_1(\alpha_1+2)p_{21}}\right]\right.$$

$$\left. -\frac{3\gamma_1-1}{\gamma_1(\gamma_1+1)}\left[1+\frac{1+4\gamma_1-5\gamma_1^2}{2\gamma_1(3\gamma_1-1)M_1^2}\right](1-a_{56})-a_{56}\frac{\bar{x}_{\text{B}}}{\bar{L}_{1\,\text{t.i.r.}}}\right\}, \tag{12}$$

where

$$\bar{x}_{\text{B}} \simeq (\gamma_{14}\sqrt{\alpha_{14}\beta_{14}}M_2-1)\left(1+\frac{\gamma_4-1}{2}\gamma_{14}\sqrt{\alpha_{14}\beta_{14}}M_2\right)^{\frac{3-\gamma_4}{2(\gamma_4-1)}} \tag{13}$$

$$\bar{t}_{\text{B}} \simeq \left(1+\frac{\gamma_4-1}{2}\gamma_{14}\sqrt{\alpha_{14}\beta_{14}}M_2\right)^{\frac{\gamma_4+1}{2(\gamma_4-1)}} \tag{14}$$

$$M_2 = \frac{p_{21}-1}{\gamma_1\sqrt{\beta_1 p_{21}(\alpha_1+p_{21})}}. \tag{15}$$

In the case $\gamma_1 = \gamma_4$, the working time can be computed by means of the exact formula presented in [4] for a shock tube with a longer high-pressure chamber than the optimal (Fig. 1).

The relative difference between the results of computing the quantities $\bar{t}_{\text{work. t. i. r.}}$ and $\bar{L}_{1\,\text{t. i. r.}}$ by means of the approximate and exact formulas is several percent and diminishes with the increase in the number M_1.

The results of computing the relative parameters $\bar{L}_{1\,\text{t. i. r.}}$, $\bar{t}_{\text{work. t. i. r.}}$ and the working time $t_{\text{work.1m}}$ of a shock tube $L_1 + L_4 = 1$ m long as a function of the shock number M_1 are presented in Fig. 3 for the helium—air gas combination.

The relationships (1), (2), (11), (12) presented can be used to estimate the initial parameters, geometric dimensions, and working time of relatively short shock tubes working with pure gases or different mixtures at moderate incident shock intensities.

Formula (1) can be extended to the case when there is a hole in the end face of the shock tube channel. However, it can be shown that the influence of the hole on the shock number M_1 is negligible in the "tailored" interface region [7].

NOTATION

x, coordinate measured from the diaphragm along the shock tube channel, m; L, length, m; t, time measured from the time of diaphragm rupture, sec; T, temperature, °K; p, pressure bar; ρ, density, kg/m^3; a, speed of sound, m/sec; u, stream velocity, m/sec; U, shock velocity, m/sec; μ, molecular weight, kg/kg·mole; M, stream or shock Mach number; γ, ratio of the specific heats; $\bar{L}$, dimensionless length, $\bar{t}$, dimensionless time.

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FLOW OF A VISCOUS TWO-TEMPERATURE NONEQUILIBRIUM IONIZED RADIATING GAS OVER BLUNT BODIES

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This paper investigates hypersonic flow of a monatomic viscous two-temperature nonequilibrium ionized radiating gas over blunt bodies. The transport coefficients are evaluated to a high-order approximation and their influence on the heat flux to the wall is analyzed.

An investigation of flow of a nonequilibrium ionized radiating gas over blunt bodies is a matter of great interest. The analogous problems were examined in [1-6] for a perfect gas, in [7, 8] (single-temperature approximation), and in [9, 10] (two-temperature approximation) for a viscous gas. However, it is suggested even in [9, 10] that the ionization reactions are frozen, radiation is absent, and the transport coefficients are calculated using very simple classical theory [11].

In this paper the problem of flow over a blunt body is posed in the most general form: the gas is regarded as viscous, heat-conducting, two-temperature, nonequilibrium-ionized, and radiating, and the transport coefficients are determined from high-order approximation theory.

The kinetic model of a gas (argon is chosen here) provides for atom—atom and electron—atom collisional ionizing reactions via an excited level

$$A + A \rightleftarrows A^* + A,\ A^* + A \rightleftarrows A^+ + e + A, \tag{1}$$

$$A + e \rightleftarrows A^* + e,\ A^* + e \rightleftarrows A^+ + 2e, \tag{2}$$

and also photon-ionization reactions with the ground level

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